Indirect Constraints and SUSY

Albert De Roeck
CERN &
Antwerp University &
IPPP Durham, UK

Oliver Buchmuller
CERN

LHC New Physics Signatures Workshop
Use of indirect constraints: EW Fit

A prominent example for the comprehensive usage of indirect constraints!

Use high precision EW measurements to:

a) Test the consistency of the SM
b) Predict the last unknown quantity in the SM: the higgs mass
Common framework

• Goal: a framework to provide consistent indirect constraints

• Collaboration of interested theorists and experimentalists
  Buchmüller, Oliver (CERN) – Exp.
  Heinemeyer, Sven (Santander) – Theo.
  Olive, Keith (Uni. of Minnesota) – Theo.
  Ronga, Frédéric (CERN) – Exp.
  Weiglein, Georg (Durham) – Theo.
  Cavanaugh, Richard (Uni. of Florida) – Exp.
  Ellis, John (CERN) – Theo.
  Isidori, Gino (INFN Frascati) – Theo.
  Paradisi, Paride (Uni. of Valencia) – Theo.

• Started at workshop on Flavour Physics in the Era of the LHC
  ⇒ See (draft) report, sec. 5.2

• Main focus of the work:
  ■ Development of a common tool for indirect constraints
  ■ Compilation (and integration) of state-of-the-art predictions
  ■ Application of the tool

Buchmuller et al., PLB 657/1-3 pp 87-94
Common framework

- Consistency
  Relies on SLHA interface
- Modularity
  Compare calculations
  Add/remove predictions
- State-of-the-art calculations
  Direct use of code from experts
  ⇒ A unique “platform” for the integration of tools

First Exercise:
Look for regions in CMSSM Space using the constraints
"Preferred" Parameter Space

Pulls from mSUGRA fit:
\( \chi^2/\text{NDF} = 17/14 ; P(\chi^2)=20\% \)

Collaboration of experimentalist and theorist:
arXiv:0707.3447

\[
\chi^2 = \sum_i \frac{(C_i - P_i)^2}{\sigma(C_i)^2 + \sigma(P_i)^2} + \sum_i \frac{(f_{\text{obs}}^{SM_i} - f_{\text{fit}}^{SM_i})^2}{\sigma(f_{\text{SM}_i})^2}
\]

Multi-parameter fit using all mSUGRA parameters. Relevant SM uncertainties like \( \Delta m_{\text{top}} \) are also considered.

\[
\Delta \chi^2 \approx 2
\]

95% Contour* 68% Contour
Preferred SUSY Parameter Space

CMS Discovery Reach for 1 fb⁻¹ (ATLAS similar)

Example of similar analyses:
- Ellis, Heinemeyer, Olive, Weber, Weiglein
  - ph/0706.0652
- Allanach, Lester, Weber
  - ph/0705.0487
- Trotta, Austin, Roszkowski
  - ph/0609126
- … there are more!

arXiv:0707.3447
95% contour obtained form a multi-parameter $\chi^2$ fit to important indirect constraints.
$\chi^2$/NDF = 17/15 - good fit
NOTE: All mSUGRA parameters are free in the fit!

“CMSSM fit clearly favors low-mass SUSY”
“best CMSSM Fit”

“LHC weather forecast”

<table>
<thead>
<tr>
<th>M0</th>
<th>M12</th>
<th>A0</th>
<th>tb</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.2</td>
<td>232.3</td>
<td>-122.4</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Ma=372 GeV; mu=336 GeV; mh=111 GeV
Higgs Mass “Prediction”

**LEP** $m_H$ constrained not used in fit

- Constrain $m_h$ to scan value;
- minimize all model parameters in each point;

$\Rightarrow$ determine error on $m_h$ prediction

**SM fit:**

- $m_H = 78^{+33}_{-24}$ GeV/$c^2$
- 12% probability at exclusion limit
  *Including theoretical uncertainty*

**CMSSM fit:**

- $m_h = 110^{+8}_{-10} \pm 3$ GeV/$c^2$
- 20% probability at exclusion limit
  *Including theoretical uncertainty*
**CMSSM vs. SM**

<table>
<thead>
<tr>
<th>Variable</th>
<th>CMSSM Measurement</th>
<th>CMSSM Fit</th>
<th>SM Measurement</th>
<th>SM Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \alpha_{\text{had}}^{(2)}(m_Z)$</td>
<td>0.02758 ± 0.00035</td>
<td>0.02774</td>
<td>0.02758 ± 0.00035</td>
<td>0.02768</td>
</tr>
<tr>
<td>$m_Z$ [GeV]</td>
<td>91.1875 ± 0.0021</td>
<td>91.1873</td>
<td>91.1875 ± 0.0021</td>
<td>91.1875</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>2.4952</td>
<td>2.4952 ± 0.0023</td>
<td>2.4952</td>
</tr>
<tr>
<td>$g_{\text{had}}^0$ [nb]</td>
<td>41.540 ± 0.037</td>
<td>41.486</td>
<td>41.540 ± 0.037</td>
<td>41.477</td>
</tr>
<tr>
<td>$R_l$</td>
<td>20.767 ± 0.025</td>
<td>20.744</td>
<td>20.767 ± 0.025</td>
<td>20.744</td>
</tr>
<tr>
<td>$A_{t,b}$</td>
<td>0.01714 ± 0.00095</td>
<td>0.01641</td>
<td>0.01714 ± 0.00095</td>
<td>0.01645</td>
</tr>
<tr>
<td>$A_t(P_t)$</td>
<td>0.1465 ± 0.0032</td>
<td>0.1479</td>
<td>0.1465 ± 0.0032</td>
<td>0.1481</td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.21629 ± 0.00066</td>
<td>0.21613</td>
<td>0.21629 ± 0.00066</td>
<td>0.21586</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.1721 ± 0.0030</td>
<td>0.1722</td>
<td>0.1721 ± 0.0030</td>
<td>0.1722</td>
</tr>
<tr>
<td>$A_{t,b}$</td>
<td>0.0992 ± 0.0016</td>
<td>0.1037</td>
<td>0.0992 ± 0.0016</td>
<td>0.1038</td>
</tr>
<tr>
<td>$A_{t,c}$</td>
<td>0.0707 ± 0.0035</td>
<td>0.0741</td>
<td>0.0707 ± 0.0035</td>
<td>0.0742</td>
</tr>
<tr>
<td>$A_b$</td>
<td>0.923 ± 0.020</td>
<td>0.935</td>
<td>0.923 ± 0.020</td>
<td>0.935</td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.670 ± 0.027</td>
<td>0.668</td>
<td>0.670 ± 0.027</td>
<td>0.668</td>
</tr>
<tr>
<td>$A_t(SLD)$</td>
<td>0.1513 ± 0.0021</td>
<td>0.1479</td>
<td>0.1513 ± 0.0021</td>
<td>0.1481</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}^{(Q_b)}$</td>
<td>0.2324 ± 0.0012</td>
<td>0.2314</td>
<td>0.2324 ± 0.0012</td>
<td>0.2314</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>80.398 ± 0.025</td>
<td>80.382</td>
<td>80.398 ± 0.025</td>
<td>80.374</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>170.9 ± 1.8</td>
<td>170.8</td>
<td>170.9 ± 1.8</td>
<td>171.3</td>
</tr>
<tr>
<td>$R(b \rightarrow s\gamma)$</td>
<td>1.13 ± 0.12</td>
<td>1.12 (N/A)</td>
<td>1.13 ± 0.12</td>
<td>1.12 (N/A)</td>
</tr>
<tr>
<td>$B_s \rightarrow \mu \mu$ [$\times 10^9$]</td>
<td>&lt; 8.00</td>
<td>0.33 (upper limit)</td>
<td>&lt; 8.00</td>
<td>0.33 (upper limit)</td>
</tr>
<tr>
<td>$\Delta \alpha_s$ [$\times 10^3$]</td>
<td>2.95 ± 0.87</td>
<td>2.95</td>
<td>2.95 ± 0.87</td>
<td>2.95</td>
</tr>
<tr>
<td>$\omega h^2$</td>
<td>0.113 ± 0.009</td>
<td>0.113</td>
<td>0.113 ± 0.009</td>
<td>0.113</td>
</tr>
</tbody>
</table>

$\chi^2/\text{ndof} = 17.0/13$ (20% prob.)

$\chi^2/\text{ndof} = 18.2/13$ (15% prob.)
New Analyses

Markov Chain Monte Carlo (MCMC):
To ensure a comprehensive mapping of the parameter space we have performed several MCMC’s with many different starting points. The shown contour is the combined result of all of them.

χ² Minima:
The overall χ² minima is determined using Minuit. The chosen starting values are determined from the results of the MCMC sampling of the parameter space.

Contours:
Contours are defined from all MCMC’s. So far, we have not performed toys to validate and refine all the 68% (blue) and 95% (red) contours but cross checks show that the contours are reliable.
# Extension of the Constraints

<table>
<thead>
<tr>
<th>Low energy observables</th>
<th>High energy EW observables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{BR}(b\to s\gamma)$</td>
<td>$\mathcal{R}_l$</td>
</tr>
<tr>
<td>$\text{BR}(b\to sll)$</td>
<td>$\mathcal{R}_b$</td>
</tr>
<tr>
<td>$\text{BR}(B_s\to\mu\mu)$</td>
<td>$\mathcal{R}_c$</td>
</tr>
<tr>
<td>$\text{BR}(B\to\tau\nu)$</td>
<td>$\mathcal{A}_{fb}(b)$</td>
</tr>
<tr>
<td>$\text{BR}(K\to\tau\nu)$</td>
<td>$\mathcal{A}_{fb}(c)$</td>
</tr>
<tr>
<td>$\text{BR}(K\to\pi\nu\nu)$</td>
<td>$\mathcal{A}_b$</td>
</tr>
<tr>
<td>$\Delta m_s/\Delta m_d$</td>
<td>$\mathcal{A}_c$</td>
</tr>
<tr>
<td>$\Delta m_s$</td>
<td>$\mathcal{A}_l(SLD)$</td>
</tr>
<tr>
<td>$\Delta m_K$</td>
<td>$\sin^2\theta_{\text{eff}}$</td>
</tr>
<tr>
<td>$g-2$</td>
<td>$m_W$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma_Z$</td>
</tr>
</tbody>
</table>

**Higgs sector observables**

| $m_h^{\text{light}}$ | $\mathcal{A}_l(SLD)$ |

**Cosmology observable**

| $\Omega h^2$ | $\mathcal{A}_l(SLD)$ |

**Improved Heavy flavour code** (particularly for high tan$\beta$)
Non-Universal Higgs Model (NUHM)

“NUHM = CMSSM but with decoupled Higgs sector at GUT scale”

\[ m_0 = m_{SQ} = m_{SL} = m_H \]

**NUHM I Model (5 Parameter)**

\[ m_0, m_{1/2}, A_0, \tan\beta, M_h [M_{hu} = M_{hd}] \]

**NUHM II Model (6 Parameter)**

\[ m_0, m_{1/2}, A_0, \tan\beta, M_{hu}, M_{hd} \]
NUHM I Contours - Part I

95% CL
68% CL
NUHM I Contours - Part II

Preliminary

95% CL
68% CL
Considered Models

**NUMH I TODAY:** Use of indirect constraints only

**CMSSM TODAY:** Use of indirect constraints only

**CMSSM 2009:** Use of indirect constraints & assumed kinematic edge measurements from LHC:

\[
\begin{align*}
(m_{ql}^2)_{\text{edge}} &= \frac{(m_{\chi_0^2}^2 - m_{RL}^2)(m_{RL}^2 - m_{\chi_1^0}^2)}{m_{RL}^2} \\
(m_{qL}^2)_{\text{edge}} &= \frac{(m_{qL}^2 - m_{\chi_0^2}^2)(m_{\chi_0^2}^2 - m_{\chi_1^0}^2)}{m_{\chi_0^2}^2} \\
(m_{qL}^2)_{\text{min}} &= \frac{(m_{qL}^2 - m_{\chi_0^2}^2)(m_{\chi_0^2}^2 - m_{RL}^2)}{m_{\chi_0^2}^2} \\
(m_{qL}^2)_{\text{max}} &= \frac{(m_{qL}^2 - m_{\chi_0^2}^2)(m_{RL}^2 - m_{\chi_1^0}^2)}{m_{RL}^2}
\end{align*}
\]

Assume 5% measurement of the edge with leptons only

Assume 10% measurements of the kinematic Quantities involving jets

[Conservative uncertainty estimates]
MSSM Higgs -95% CL

- NUMH1 TODAY (95% CL) Prel.
- CMSSM TODAY (95% CL)
- CMSSM 2009 (95% CL)
MSSM Higgs - 68% CL

- NUMH1 TODAY (68% CL) prel.
- CMSSM TODAY (68% CL)
- CMSSM 2009 (68% CL)

Preliminary
Dark Matter

Direct detection of WIMP (LSP) Dark Matter

Sensitivity Plot:
WIMP(LSP) Mass vs. $\sigma_p^{SI}$

$\sigma_p^{SI}$: spin-independent dark matter WIMP elastic scattering cross section on a free proton.

A convenient way to illustrate direct and indirect WIMP searches.
WIMP Sensitivity Plot

Contours:
- NUMH1 TODAY (95% CL) prel.
- CMSSM TODAY (95% CL)
- CMSSM 2009 (95% CL)
Spectrum Comparison

“best CMSSM Fit”

<table>
<thead>
<tr>
<th>M0</th>
<th>M12</th>
<th>A0</th>
<th>tb</th>
<th>MA</th>
<th>mu</th>
<th>mh</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.2</td>
<td>232.3</td>
<td>-122.4</td>
<td>6.9</td>
<td>372 GeV</td>
<td>336 GeV</td>
<td>111 GeV</td>
</tr>
</tbody>
</table>

“best NUHM Fit”

<table>
<thead>
<tr>
<th>M0</th>
<th>M12</th>
<th>A0</th>
<th>tb</th>
<th>Mhd²</th>
<th>Mhu²</th>
<th>MA</th>
<th>mu</th>
<th>mh</th>
</tr>
</thead>
<tbody>
<tr>
<td>101.9</td>
<td>208.1</td>
<td>-523.1</td>
<td>6.7</td>
<td>183000</td>
<td>72300</td>
<td>220 GeV</td>
<td>460 GeV</td>
<td>113 GeV</td>
</tr>
</tbody>
</table>
Conclusions

- Modular framework for comparison (~) in place
  - Allows to study “preference” of new physics phase space
  - Allows to study consistency between new signals and precision data
  - Expect to become important for the interpretation of potential discoveries, eg dark matter and heavy Higgs constraints

- Early SUSY discovery@LHC “preferred”

- Higgs perhaps already seen by LEP 😊

- This is an open project. Collaborators welcome

- Next steps could/will include
  - More systematic study of the individual effect of different variables
  - More systematic study on the uncertainties (eg. sparticle spectrum)
  - More general SUSY models
  - Other than SUSY BSM
Backup
Dark Matter - WIMP(LSP) contours

- NUMH1 TODAY (95% CL)
- CMSSM TODAY (95% CL)
- CMSSM 2009 (95% CL)

Cross-section [cm$^2$] (normalised to nucleon)

Log$_{10}$($\sigma_p$)

WIMP Mass [GeV/c$^2$]

WIMP Mass [GeV]
Define “hypothetical scenario” with \( m_h = 70 \text{ GeV} \) (black line). Errors on indirect constraints are kept but constraints are varied to be compatible with hypothesis.

**Conclusion:**

“Narrowsness” of the ellipse is NOT a property of a particular scenario but rather determined by the errors of the indirect constraints and the general model properties of the Higgs sector in the CMSSM.
- Multi-parameter $\chi^2$ fit:

$$\chi^2 = \sum_i \frac{(C_i - P_i)^2}{\sigma(C_i)^2 + \sigma(P_i)^2} + \sum_i \frac{(f_{SM; i}^{obs} - f_{SM; i}^{fit})^2}{\sigma(f_{SM; i})^2}$$

- fitting for all CMSSM parameters: $M_0$, $M_{1/2}$, $A_0$, $\tan \beta$;
- including relevant SM uncertainties (e.g. $m_{top}$);

→ details in O. Buchmüller et al., arXiv:0707.3447 [hep-ph]

Natural extension of J. Ellis et al., arXiv:0706.0652 [hep-ph]
From fits on 2000 pseudo-experiments

- overall preferred minimum at low $\tan \beta$, low squark mass;
- less preferred region at high $\tan \beta$, higher squark mass;
- consistent with previous studies.

Note: includes limit from LEP

⇒ Turn to fit without limit on $m_h$
assessing preferred $m_h$ value
in CMSSM
CMSSM

Constrain soft-breaking parameters at the GUT scale

CMSSM - a very constraint model:

- Unification of the gaugino [bino, wino and gluino] masses:

\[ M_1(M_{\text{GUT}}) = M_2(M_{\text{GUT}}) = M_3(M_{\text{GUT}}) \equiv m_{1/2} \]

- Universal scalar [i.e. sfermion and Higgs boson] masses [\( i \) is the generation index]:

\[
\begin{align*}
M_{\tilde{Q}_i}(M_{\text{GUT}}) &= M_{\tilde{U}_i}(M_{\text{GUT}}) = M_{\tilde{D}_i}(M_{\text{GUT}}) = M_{\tilde{L}_i}(M_{\text{GUT}}) = M_{\tilde{E}_i}(M_{\text{GUT}}) \\
&= M_{H_\pm}(M_{\text{GUT}}) = M_{H^0}(M_{\text{GUT}}) \equiv m_0
\end{align*}
\]

- Universal trilinear couplings:

\[ A_{ij}^u(M_{\text{GUT}}) = A_{ij}^d(M_{\text{GUT}}) = A_{ij}^l(M_{\text{GUT}}) \equiv A_0 \delta_{ij} \]

Free Parameters:

\[ m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu) \]

All the soft SUSY breaking parameters at the weak scale are obtained through Renormalization Group Equations (RGE’s)

- \( m_0 \): common scalar mass at GUT
- \( m_{1/2} \): the common gaugino mass at GUT
- \( \tan\beta \): \( <H_u>/<H_d> \)
- \( A_0 \): common (scalar)\(^3\) coupling
- \( \text{sign}(\mu) \): Higgs mass term
Constrain $m_h$ to scan value;

minimize all model parameters in each point;

$\Rightarrow$ determine error on $m_h$ prediction

**SM fit:**

- $m_H = 78^{+33}_{-24}$ GeV/$c^2$
- 12% probability at exclusion limit
  \textit{Including theoretical uncertainty}

**CMSSM fit:**

- $m_h = 110^{+8}_{-10} \pm 3$ GeV/$c^2$
- 20% probability at exclusion limit
  \textit{Including theoretical uncertainty}
New list of constraints

**Flavour code**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(b \to s\gamma)$</td>
<td>$1.13 \pm 0.12$</td>
</tr>
<tr>
<td>$R(\Delta m_b)$</td>
<td>$1.040 \pm 0.340$</td>
</tr>
<tr>
<td>$B_s \to \mu\mu$</td>
<td>&lt; 4.7000</td>
</tr>
<tr>
<td>$R(B \to \tau\nu)$</td>
<td>$1.07 \pm 0.42$</td>
</tr>
<tr>
<td>$R(B_s \to X_s\ell\ell)$</td>
<td>$0.99 \pm 0.32$</td>
</tr>
<tr>
<td>$R(K \to \tau\nu)$</td>
<td>$0.992 \pm 0.017$</td>
</tr>
<tr>
<td>$R(\Delta m_K)$</td>
<td>$0.880 \pm 0.320$</td>
</tr>
<tr>
<td>$R(K \to \pi\nu\nu)$</td>
<td>&lt; 4.5</td>
</tr>
<tr>
<td>$B(B_d \to \ell\ell)$</td>
<td>&lt; 2.30</td>
</tr>
<tr>
<td>$R(\Delta m_{\mu}/\Delta m_d)$</td>
<td>$1.00 \pm 0.08$</td>
</tr>
</tbody>
</table>

**SUSY-POPE**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta a_{\mu}^{(S)}(m_Z)$</td>
<td>$0.02758 \pm 0.00035$</td>
</tr>
<tr>
<td>$m_Z$ [GeV]</td>
<td>$91.1875 \pm 0.0021$</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>$2.4952 \pm 0.0023$</td>
</tr>
<tr>
<td>$c_{\text{had}}^\phi$ [nb]</td>
<td>$41.540 \pm 0.037$</td>
</tr>
<tr>
<td>$R_\gamma$</td>
<td>$20.767 \pm 0.025$</td>
</tr>
<tr>
<td>$A_{tb}^{0,1}$</td>
<td>$0.01714 \pm 0.00095$</td>
</tr>
<tr>
<td>$A_t(P_t)$</td>
<td>$0.1465 \pm 0.0032$</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$0.21629 \pm 0.00066$</td>
</tr>
<tr>
<td>$R_c$</td>
<td>$0.172 \pm 0.003$</td>
</tr>
<tr>
<td>$A_{tb}^{0,b}$</td>
<td>$0.0992 \pm 0.0016$</td>
</tr>
<tr>
<td>$A_{tb}^{0,c}$</td>
<td>$0.0707 \pm 0.0035$</td>
</tr>
<tr>
<td>$A_b$</td>
<td>$0.923 \pm 0.020$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$0.670 \pm 0.027$</td>
</tr>
<tr>
<td>$A_t(SLD)$</td>
<td>$0.1513 \pm 0.0021$</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}(Q_{tb})$</td>
<td>$0.2324 \pm 0.0012$</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>$80.398 \pm 0.025$</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>$170.9 \pm 1.8$</td>
</tr>
</tbody>
</table>

FeynHiggs

MicrOMEGAs

$+ m_h > (115 \pm 1.1 \pm 3) \text{ GeV from FeynHiggs}$